Homework 4
Due 9/25/2009

1. Consider an n-node binary tree such that the for every node in the tree, the height of the left and right subtrees of that node differ by at most 1 (the AVL balance requirement). Prove that the height of such a tree has the following bound:

$$\text{height} \leq 2 \log n$$

   As discussed in class, inserting and searching in binary search trees takes $O(height)$ time. As an AVL binary search tree maintains height balance, this theorem shows that the AVL tree can perform insertions and searchs in $O(\log n)$ time. (Suggestion: Use induction)

2. Prove that $\log n! = \Theta(n \log n)$.

3. 4.4, 4.5, 4.6, 4.9

4. For each of the following pairs of functions either $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$. Determine which relationships are true for each pair.

   (a) $f(n) = \log n^2$; $g(n) = \log n + 5$
   (b) $f(n) = \sqrt{n}$; $g(n) = \log n^2$
   (c) $f(n) = \log^2 n$; $g(n) = \log n$
   (d) $f(n) = n$; $g(n) = \log^2 n$
   (e) $f(n) = n \log n + n$; $g(n) = \log n$
   (f) $f(n) = 10$; $g(n) = \log 10$
   (g) $f(n) = 2^n$; $g(n) = 10n^2$
   (h) $f(n) = 2^n$; $g(n) = 3^n$

5. Find the big-Oh solution to the following recurrence equations.

   (a) $T(n) = T(\frac{9}{10}n) + n$
   (b) T(n) = $16T(\frac{n}{4}) + n^2$
   (c) T(n) = $2T(\frac{n}{4}) + \sqrt{n}$